

A Note of Extended Proca Equations and Superconductivity

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It has been known for quite long time that the electrodynamics of Maxwell equations can be extended and generalized further into Proca equations. The implications of introducing Proca equations include an alternative description of superconductivity, via extending London equations. In the light of another paper suggesting that Maxwell equations can be written using quaternion numbers, then we discuss a plausible extension of Proca equation using biquaternion number. Further implications and experiments are recommended.

1 Introduction

It has been known for quite long time that the electrodynamics of Maxwell equations can be extended and generalized further into Proca equations, to become electrodynamics with finite photon mass [11]. The implications of introducing Proca equations include description of superconductivity, by extending London equations [20]. In the light of another paper suggesting that Maxwell equations can be generalized using quaternion numbers [3, 7], then we discuss a plausible extension of Proca equations using biquaternion number. It seems interesting to remark here that the proposed extension of Proca equations by including quaternion differential operator is merely the next logical step considering already published suggestion concerning the use of quaternion differential operator in electromagnetic scattering problems [7, 13, 14]. This is called Moisil-Theodoresco operator (see also Appendix A).

2 Maxwell equations and Proca equations

In a series of papers, Lehnert argued that the Maxwell picture of electrodynamics shall be extended further to include a more “realistic” model of the *non-empty vacuum*. In the presence of electric space charges, he suggests a general form of the Proca-type equation [11]:

$$\left(\frac{1}{c^2} \frac{\partial}{\partial t^2} - \nabla^2 \right) A_\mu = \mu_0 J_\mu, \quad \mu = 1, 2, 3, 4. \quad (1)$$

Here $A_\mu = (A, i\phi/c)$, where A and ϕ are the magnetic vector potential and the electrostatic potential in three-space, and:

$$J_\mu = (j, ic\bar{\phi}). \quad (2)$$

However, in Lehnert [11], the right-hand terms of equations (1) and (2) are now given a new interpretation, where $\bar{\phi}$ is the nonzero electric charge density in the vacuum, and j stands for an associated three-space current-density.

The background argument of Proca equations can be summarized as follows [6]. It was based on known definition of derivatives [6, p. 3]:

$$\left. \begin{aligned} \partial^\mu &= \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = (\partial^0; -\nabla) \\ \partial_\mu &= \frac{\partial}{\partial x^\mu} = (\partial^0; \nabla) \end{aligned} \right\}, \quad (3)$$

$$\partial_\mu a^\mu = \frac{\partial a^0}{\partial t} + \nabla \vec{a}, \quad (4)$$

$$\partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \partial_0^2 - \nabla^2 = \partial^\mu \partial_\mu, \quad (5)$$

where ∇^2 is Laplacian and $\partial_\mu \partial^\mu$ is d'Alembertian operator. For a massive vector boson (spin-1) field, the Proca equation can be written in the above notation [6, p. 7]:

$$\partial_\mu \partial^\mu A^\nu - \partial^\nu (\partial_\mu A^\mu) + m^2 A^\nu = j^\nu. \quad (6)$$

Interestingly, there is also a neat link between Maxwell equations and quaternion numbers, in particular via the *Moisil-Theodoresco D operator* [7, p. 570] and [13]:

$$D = i_1 \frac{\partial}{\partial x_1} + i_2 \frac{\partial}{\partial x_2} + i_3 \frac{\partial}{\partial x_3}. \quad (7)$$

There are also known links between Maxwell equations and Einstein-Mayer equations [8]. Therefore, it seems plausible to extend further the Maxwell-Proca equations to biquaternion form too; see also [9, 10] for links between Proca equation and Klein-Gordon equation. For further theoretical description on the links between biquaternion numbers, Maxwell equations, and unified wave equation, see Appendix A.

3 Proca equations and superconductivity

In this regards, it has been shown by Sternberg [20], that the classical *London equations* for superconductors can be written in differential form notation and in relativistic form, where

they yield the Proca equations. In particular, the field itself acts as its own charge carrier [20].

Similarly in this regards, in a recent paper Tajmar has shown that superconductor equations can be rewritten in terms of Proca equations [21]. The basic idea of Tajmar appears similar to Lehnert's extended Maxwell theory, i.e. to include *finite photon mass* in order to explain superconductivity phenomena. As Tajmar puts forth [21]:

"In quantum field theory, superconductivity is explained by a *massive photon*, which acquired mass due to gauge symmetry breaking and the Higgs mechanism. The wavelength of the photon is interpreted as the London penetration depth. With a nonzero photon mass, the usual Maxwell equations transform into the so-called Proca equations which will form the basis for our assessment in superconductors and are only valid for the superconducting electrons."

Therefore the basic Proca equations for superconductor will be [21, p.3]:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (8)$$

and

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \frac{1}{\lambda^2} \vec{A}. \quad (9)$$

The Meissner effect is obtained by taking curl of equation (9). For non-stationary superconductors, the same equation (9) above will yield second term, called London moment.

Another effects are recognized from the finite Photon mass, i.e. the photon wavelength is then interpreted as the London penetration depth and leads to a photon mass about 1/1000 of the electron mass. This furthermore yields the Meissner-Ochsenfeld effect (shielding of electromagnetic fields entering the superconductor) [22].

Nonetheless, the use of Proca equations have some known problems, i.e. it predicts that a charge density rotating at angular velocity should produce huge magnetic fields, which is *not observed* [22]. One solution of this problem is to recognize that the value of photon mass containing charge density is different from the one in free space.

4 Biquaternion extension of Proca equations

Using the method we introduced for Klein-Gordon equation [2], then it is possible to generalize further Proca equations (1) using biquaternion differential equations, as follows:

$$(\diamond \bar{\diamond}) A_\mu - \mu_0 J_\mu = 0, \quad \mu = 1, 2, 3, 4, \quad (10)$$

where (see also Appendix A):

$$\begin{aligned} \diamond = \nabla^q + i\nabla^q = & \left(-i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) + \\ & + i \left(-i \frac{\partial}{\partial T} + e_1 \frac{\partial}{\partial X} + e_2 \frac{\partial}{\partial Y} + e_3 \frac{\partial}{\partial Z} \right). \end{aligned} \quad (11)$$

Another way to generalize Proca equations is by using its standard expression. From d'Alembert wave equation we get [6]:

$$\left(\frac{1}{c^2} \frac{\partial}{\partial t^2} - \nabla^2 \right) A_\mu = \mu_0 J_\mu, \quad \mu = 1, 2, 3, 4, \quad (12)$$

where the solution is Liennard-Wiechert potential. Then the Proca equations are [6]:

$$\left[\left(\frac{1}{c^2} \frac{\partial}{\partial t^2} - \nabla^2 \right) + \left(\frac{m_p c}{\hbar} \right)^2 \right] A_\mu = 0, \quad \mu = 1, 2, 3, 4, \quad (13)$$

where m is the photon mass, c is the speed of light, and $\hbar$ is the reduced Planck constant. Equation (13) and (12) imply that photon mass can be understood as charge density:

$$J_\mu = \frac{1}{\mu_0} \left(\frac{m_p c}{\hbar} \right)^2. \quad (14)$$

Therefore the "biquaternionic" extended Proca equations (13) become:

$$\left[\diamond \bar{\diamond} + \left(\frac{m_p c}{\hbar} \right)^2 \right] A_\mu = 0, \quad \mu = 1, 2, 3, 4. \quad (15)$$

The solution of equations (10) and (12) can be found using the same computational method as described in [2].

Similarly, the generalized structure of the wave equation in electrodynamics — *without* neglecting the finite photon mass (Lehnert-Vigier) — can be written as follows (instead of eq. 7.24 in [6]):

$$\left[\diamond \bar{\diamond} + \left(\frac{m_p c}{\hbar} \right)^2 \right] A_\mu^a = R A_\mu^a, \quad \mu = 1, 2, 3, 4. \quad (16)$$

It seems worth to remark here that the method as described in equation (15)-(16) or ref. [14] is not the only possible way towards generalizing Maxwell equations. Other methods are available in literature, for instance by using topological geometrical approach [16, 17].

Nonetheless further experiments are recommended in order to verify this proposition [25, 26]. One particular implication resulted from the introduction of biquaternion differential operator into the Proca equations, is that it may be related to the notion of "active time" introduced by Paine & Pensinger sometime ago [15]; the only difference here is that now the time-evolution becomes nonlinear because of the use of 8-dimensional differential operator.

5 Plausible new gravitomagnetic effects from extended Proca equations

While from Proca equations one can expect to observe gravitational London moment [4, 24] or other peculiar gravitational shielding effect unable to predict from the framework of General Relativity [5, 18, 24], one can expect to derive new gravitomagnetic effects from the proposed extended Proca equations using the biquaternion number as described above.

Furthermore, another recent paper [1] has shown that given the finite photon mass, it would imply that if m is due to a Higgs effect, then the *Universe is effectively similar to a Superconductor*. This may support De Matos's idea of dark energy arising from superconductor, in particular via *Einstein-Proca* description [1, 5, 18].

It is perhaps worth to mention here that there are some indirect observations [1] relying on the effect of Proca energy (assumed) on the galactic plasma, which implies the limit:

$$m_A = 3 \times 10^{-27} \text{ eV}. \quad (17)$$

Interestingly, in the context of cosmology, it can be shown that Einstein field equations with cosmological constant are approximated to the second order in the perturbation to a flat background metric [5]. Nonetheless, further experiments are recommended in order to verify or refute this proposition.

6 Some implications in superconductivity research

We would like to mention the Proca equation in the following context. Recently it was hypothesized that the creation of superconductivity at room temperature may be achieved by a resonance-like interaction between an everywhere present background field and a special material having the appropriate crystal structure and chemical composition [12]. According to Global Scaling, a new knowledge and holistic approach in science, the everywhere present background field is given by oscillations (standing waves) in the universe or physical vacuum [12].

The just mentioned hypothesis how superconductivity at room temperature may come about, namely by a resonance-like interaction between an everywhere present background field and a special material having the appropriate crystal structure and chemical composition, seems to be supported by a statement from the so-called ECE Theory which is possibly related to this hypothesis [12]:

“... One of the important practical consequences is that a material can become a superconductor by absorption of the inhomogeneous and homogeneous currents of ECE space-time ...” [6].

This is a quotation from a paper with the title “ECE Generalizations of the d'Alembert, Proca and Superconductivity Wave Equations ...” [6]. In that paper the Proca equation is derived as a special case of the ECE field equations.

These considerations raises the interesting question about the relationship between (a possibly new type of) superconductivity, space-time, an everywhere-present background field, and the description of superconductivity in terms of the Proca equation, i.e. by a massive photon which acquired mass by symmetry breaking. Of course, how far these suggestions are related to the physical reality will be decided by further experimental and theoretical studies.

7 Concluding remarks

In this paper we argue that it is possible to extend further Proca equations for electrodynamics of superconductivity to biquaternion form. It has been known for quite long time that the electrodynamics of Maxwell equations can be extended and generalized further into Proca equations, to become electrodynamics with finite photon mass. The implications of introducing Proca equations include description of superconductivity, by extending London equations. Nonetheless, further experiments are recommended in order to verify or refute this proposition.

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Appendix A: Biquaternion, Maxwell equations and unified wave equation [3]

In this section we're going to discuss Ulrych's method to describe unified wave equation [3], which argues that it is possible to define a unified wave equation in the form [3]:

$$D\phi(x) = m_\phi^2 \cdot \phi(x), \quad (A.1)$$

where unified (wave) differential operator D is defined as:

$$D = [(P - qA)_\mu (\bar{P} - qA)^\mu]. \quad (A.2)$$

To derive Maxwell equations from this unified wave equation, he uses free photon expression [3]:

$$DA(x) = 0, \quad (A.3)$$

where potential $A(x)$ is given by:

$$A(x) = A^0(x) + jA^1(x), \quad (A.4)$$

and with electromagnetic fields:

$$E^i(x) = -\partial^0 A^i(x) - \partial^i A^0(x), \quad (A.5)$$

$$B^i(x) = \epsilon^{ijk} \partial_j A_k(x). \quad (A.6)$$

Inserting these equations (A.4)-(A.6) into (A.3), one finds Maxwell electromagnetic equation [3]:

$$\begin{aligned} & -\nabla \bullet E(x) - \partial^0 C(x) + i j \nabla \bullet B(x) - \\ & - j(\nabla x B(x) - \partial^0 E(x) - \nabla C(x)) - \\ & - i(\nabla x E(x) + \partial^0 B(x)) = 0. \end{aligned} \quad (A.7)$$

For quaternion differential operator, we define quaternion Nabla operator:

$$\begin{aligned} \nabla^q & \equiv c^{-1} \frac{\partial}{\partial t} + \left(\frac{\partial}{\partial x} \right) i + \left(\frac{\partial}{\partial y} \right) j + \left(\frac{\partial}{\partial z} \right) k = \\ & = c^{-1} \frac{\partial}{\partial t} + \vec{i} \cdot \vec{\nabla}. \end{aligned} \quad (A.8)$$

And for biquaternion differential operator, we may define a diamond operator with its conjugate [3]:

$$\diamond \tilde{\diamond} \equiv \left(c^{-1} \frac{\partial}{\partial t} + c^{-1} i \frac{\partial}{\partial t} \right) + \{ \vec{\nabla} \}^* \quad (\text{A.9})$$

where Nabla-star-bracket operator is defined as:

$$\begin{aligned} \{ \vec{\nabla} \}^* \equiv & \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial X} \right) i + \\ & + \left(\frac{\partial}{\partial y} + i \frac{\partial}{\partial Y} \right) j + \left(\frac{\partial}{\partial z} + i \frac{\partial}{\partial Z} \right) k. \end{aligned} \quad (\text{A.10})$$

In other words, equation (A.9) can be rewritten as follows:

$$\begin{aligned} \diamond \tilde{\diamond} \equiv & \left(c^{-1} \frac{\partial}{\partial t} + c^{-1} i \frac{\partial}{\partial T} \right) + \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial X} \right) i + \\ & + \left(\frac{\partial}{\partial y} + i \frac{\partial}{\partial Y} \right) j + \left(\frac{\partial}{\partial z} + i \frac{\partial}{\partial Z} \right) k. \end{aligned} \quad (\text{A.11})$$

From this definition, it shall be clear that there is neat link between equation (A.11) and the Moisil-Theodoresco D operator, i.e. [5, p. 570]:

$$\begin{aligned} \diamond \tilde{\diamond} \equiv & \left(c^{-1} \frac{\partial}{\partial t} + c^{-1} i \frac{\partial}{\partial T} \right) + (D_{xi} + i D_{xi}) = \\ = & \left(c^{-1} \frac{\partial}{\partial t} + c^{-1} i \frac{\partial}{\partial T} \right) + \left[i_1 \frac{\partial}{\partial x_1} + i_2 \frac{\partial}{\partial x_2} + i_3 \frac{\partial}{\partial x_3} \right] + \\ & + i \left[i_1 \frac{\partial}{\partial X_1} + i_2 \frac{\partial}{\partial X_2} + i_3 \frac{\partial}{\partial X_3} \right]. \end{aligned} \quad (\text{A.12})$$

In order to define biquaternionic representation of Maxwell equations, we could extend Ulrych's definition of *unified differential operator* [3,19,23] to its biquaternion counterpart, by using equation (A.2) and (A.11), to become:

$$\{ D \}^* \equiv \left[\left(\{ P \}^* - q \{ A \}^* \right)_\mu \left(\{ \tilde{P} \}^* - q \{ A \}^* \right)^\mu \right], \quad (\text{A.13})$$

or by definition $P = -i \hbar \nabla$, equation (A.12) could be written as:

$$\{ D \}^* \equiv \left[\left(-\hbar \{ \vec{\nabla} \}^* - q \{ A \}^* \right)_\mu \left(-\hbar \{ \vec{\nabla} \}^* - q \{ A \}^* \right)^\mu \right], \quad (\text{A.14})$$

where each component is now defined in term of biquaternionic representation. Therefore the biquaternionic form of the *unified wave equation* [3] takes the form:

$$\{ D \}^* \phi(x) = m_\phi^2 \cdot \phi(x), \quad (\text{A.15})$$

which is a wave equation for massive electrodynamics, quite similar to Proca representation.

Now, biquaternionic representation of free photon fields could be written as follows:

$$\{ D \}^* A(x) = 0. \quad (\text{A.16})$$

References

- Adelberger E., Dvali G., and Gruzinov A. Photon-mass bound destroyed by vortices. *Phys. Rev. Lett.*, 2007, v. 98, 010402.
- Christianto V. and Smarandache F. Numerical solution of radial biquaternion Klein-Gordon equation. *Progress in Physics*, 2008, v.1.
- Christianto V. *Electronic J. Theor. Physics*, 2006, v. 3, no. 12.
- De Matos C. J. arXiv: gr-qc/0607004; Graviton-photon, superconductor and hyperdrives. <http://members.tripod.com/datheoretical1/warptohyperdrives.html>
- De Matos C.J. arXiv: gr-qc/060911.
- Evans M.W. ECE generalization of the d'Alembert, Proca and superconductivity wave equations: electric power from ECE space-time. §7.2; <http://aias.us/documents/uft/a51stpaper.pdf>
- Kravchenko V.V. and Oviedo H. On quaternionic formulation of Maxwell's equations for chiral media and its applications. *J. for Analysis and its Applications*, 2003, v. 22, no. 3, 570.
- Kravchenko V.G. and Kravchenko V.V. arXiv: math-ph/0511092.
- Jakubsky V. and Smejkal J. A positive definite scalar product for free Proca particle. arXiv: hep-th/0610290.
- Jakubsky V. *Acta Polytechnica*, 2007, v. 47, no. 2–3.
- Lehnert B. Photon physics of revised electromagnetics. *Progress in Physics*, 2006, v. 2.
- Lichtenberg F. Presentation of an intended research project: searching for room temperature superconductors. August, 2008, <http://www.sciprint.org>; http://podtime.net/sciprint/fm/uploads/files/1218979173Searching_for_Room_Temperature_Superconductors.pdf
- Kmelnytskaya K., Kravchenko V.V., and Rabinovich V.S. arXiv: math-ph/0206013.
- Kmelnytskaya K. and Kravchenko V.V. arXiv: 0706.1744.
- Paine D.A. and Pensinger W.L. *Int. J. Quantum Chem.*, 1979, v.15, 3; <http://www.geocities.com/moonhoabinh/ithapapers/hydrothermo.html>
- Olkhov O.A. Geometrization of classical wave fields. arXiv: 0801.3746.
- Olkhov O.A. *Zh. Fiz. Khim.*, 2002, v. 21, 49; arXiv: hep-th/0201020.
- Poenaru D. A. Proca (1897–1955). arXiv: physics/0508195; <http://th.physik.uni-frankfurt.de/~poenaru/PROCA/Proca.pdf>
- Ulrych S. arXiv: physics/0009079.
- Sternberg S. On the London equations. *PNAS*, 1992, v. 89, no. 22, 10673–10675.
- Tajmar M. Electrodynamics in superconductors explained by Proca equations. arXiv: 0803.3080.
- Tajmar M. and De Matos C.J. arXiv: gr-qc/0603032.
- Yefremov A., Smarandache F. and Christianto V. Yang-Mills field from quaternion space geometry, and its Klein-Gordon representation. *Progress in Physics*, 2007, v. 3.
- Gravitational properties of superconductors. http://functional-materials.at/rd/rd_spa_gravitationalproperties_de.html
- Magnetism and superconductivity observed to exist in harmony. Aug. 28, 2008, <http://www.physorg.com/news139159195.html>
- Room temperature superconductivity. Jul. 8, 2008, <http://www.physorg.com/news134828104.html>